M/M/1/K RETRIAL QUEUEING MODEL WITH CONTROLLABLE ARRIVAL RATES, BALKING AND RENEGING

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Abstract

In this paper, a finite capacity single server interdependent retrial queueing model with controllable arrival rates, balking and reneging is considered. The steady state solutions and the system characteristics are derived and analyzed for this model. Some particular cases of the model have been discussed. Numerical results are given for better understanding and relevant conclusion is presented.

Keywords

retrial queue; interdependent primary arrival and service processes; finite capacity; reneging; balking;

1.0 Introduction

Retrial queueing systems differ from conventional queueing systems in that customers arriving to a server station and finding all servers unavailable enter a retrial orbit (or source of repeated calls) instead of a normal queue. They remain there for a random amount of time (usually exponentially distributed), and then check to see if a server is available. If a server is available, they enter service immediately; otherwise they return to the orbit and wait again. In the meantime, a new or primary customer can arrive to the system and obtain service if a server is free. These queuing models arise in the stochastic modeling of many communication protocols, local area networks , teletraffic theory and daily life situations. There are so many applications found for retrial queues in science and engineering streams. Abandonment happens when a subscriber's call becomes rejected and the subscriber gets impatient and gives up after a certain time without getting service. The model studied in this paper not only takes into account retrials due to congestion but also considers the effects of balking and reneging of customers.

In this paper, the M/M/1/K interdependent retrial queueing model with controllable arrival rates, balking and reneging customers is considered. In section 2, the description of the model is given stating the relevant postulates, the steady state equations and the characteristics of the model are derived. In section 3, numerical results are illustrated.

2.0 Description of The Model

Consider a single server finite capacity finite source retrial queueing system in which primary customers arrive according to the Poisson flow of rate λ_0 and λ_1, service times are exponentially distributed with rate μ. If a primary customer finds some server free, he instantly occupies it and leaves the system after service. Otherwise, if the server is busy, at the time of arrival of a primary call then with probability $H_1 \geq 0$ the arriving customer enters an orbit and repeats his demand after an exponential time with rate θ. Thus the Poisson flow of repeated call follow the retrial policy where the repetition times of each customer is assumed to be independent and exponentially distributed.

If an incoming repeated call finds the line free, it is served and leaves the system after service, while the source which produced this repeated call disappears. Otherwise, if the server is occupied at the time of a repeated call arrival with probability $(1-H_2)$ the source leaves the system without service. Each customer upon arriving in the queue will wait a certain length of time (reneging time) for his service to begin. If it has not begun by then, he will get impatient and may leave the queue without getting service with probability p and may remain in the queue for his service with probability (q = 1-p). The reneging times follow exponential distribution with parameter α.

It is assumed that the primary arrival process $[X_1(t)]$ and the service process $[X_2(t)]$ of the systems are correlated and follow a bivariate Poisson process given by

$$P(X_1=x_1,X_2=x_2;t) = e^{-(\lambda_i + \mu-\varepsilon)t} \frac{\sum_{j=0}^{\min(x_1 x_2)} \varepsilon(t)^j \ (\lambda_i-\varepsilon)t^{x_1-j} (\mu-\varepsilon)t^{x_2-j}}{j!\,(x_1-j)!\,(x_2-j)!}$$

$x_1, x_2 = 0,1,2,\ldots\ldots,\lambda_i , \mu < 0,\ i = 0,1;$

with parameters $\lambda_0, \lambda_1, \mu_n$ and ε as mean faster rate of primary arrivals, mean slower rate of primary arrivals, mean service rate and mean dependence rate (covariance between the primary arrival and service processes) respectively.
The process N(t),C(t) ;$t \geq 0$ forms a Markov chain with state space $(n,c)|n = \{0,1,2,\ldots..r-1,r,r+1,\ldots.R-1,R,R+1\ldots.K\}$,$c=\{0,1\}$. Let C and N be the numbers of customers in the service facility and in the orbit, respectively, in steady state.

2.1 Steady State Equation

We observe that only $P_{0,n,0}$ and $P_{1,n,0}$ exists when $n=0,1,2,\ldots\ldots.., r-1,r$; $P_{0,n,0}, P_{1,n,0}, P_{0,n,1}$ and $P_{1,n,1}$ exist when $n=r+1,r+2,\ldots\ldots, R-2,R-1$; $P_{0,n,1}$ and $P_{1,n,1}$ exists when $n=R,R+1,\ldots.. , K$. Further $P_{0,n,0}= P_{1,n,0} = P_{0,n,1} = P_{1,n,1} = 0$ if $n>K$.

The steady state equations are

$$-(\lambda_0-\varepsilon)P_{0,0,0}+(\mu-\varepsilon)P_{1,0,0}=0 \quad (1)$$

$$-[H_1(\lambda_0-\varepsilon)+(\mu-\varepsilon)]P_{1,0,0}+(\lambda_0-\varepsilon)P_{0,0,0}+\theta P_{0,1,0}+\theta(1-H_2)P_{1,1,0}=0 \quad (2)$$

$$-[(\lambda_0-\varepsilon)+n\theta]P_{0,n,0}+(\mu-\varepsilon)P_{1,n,0}=0 \quad (3)$$

$$-[H_1(\lambda_0-\varepsilon)+(\mu-\varepsilon)+n\theta(1-H_2)+(n-1)\alpha p]P_{1,n,0}+[(\lambda_0-\varepsilon)]P_{0,n,0}+$$
$$[H_1(\lambda_0-\varepsilon)]P_{1,n-1,0}+(n+1)\theta P_{0,n+1,0}+[(n+1)\theta(1-H_2)+n\alpha p]P_{1,n+1,0}=0,$$
$$n=1,2,3,\ldots..,r-1 \quad (4)$$

$$-[(\lambda_0-\varepsilon)+r\theta]P_{0,r,0}+(\mu-\varepsilon)P_{1,r,0}=0 \quad (5)$$

$$-[H_1(\lambda_0-\varepsilon)+(\mu-\varepsilon)+r\theta(1-H_2)+(r-1)\alpha p]P_{1,r,0}+[(\lambda_0-\varepsilon)]P_{0,r,0}+$$

$[H_1(\lambda_0 - \varepsilon)]\ P_{1,r-1,0} + (r+1)\theta\ P_{0,r+1,0} + [(r+1)\theta(1-H_2)+r\alpha p]\ P_{1,r+1,0} + (r+1)\theta\ P_{0,r+1,1} +$
$[(r+1)\theta(1-H_2)+r\alpha p]P_{1,r+1,1}=0$ (6)

$-[(\lambda_0 - \varepsilon)+n\theta]P_{0,n,0}+(\mu - \varepsilon)P_{1,n,0}=0$ (7)

$-[H_1(\lambda_0 - \varepsilon)+(\mu - \varepsilon)+n\theta(1 - H_2) + (n-1)\alpha p]\ P_{1,n,0} + [(\lambda_0 - \varepsilon)]\ P_{0,n,0} +$
$[H_1(\lambda_0 - \varepsilon)]\ P_{1,n-1,0} + (n+1)\theta\ P_{0,n+1,0} + [(n+1)\theta(1-H_2)+n\alpha p]\ P_{1,n+1,0}=0,$
$n = r+1, r+2,\ldots\ldots,R-2$ (8)

$-[(\lambda_0 - \varepsilon)+(R-1)\theta]P_{0,R-1,0}+(\mu - \varepsilon)P_{1,R-1,0}=0$ (9)

$-[H_1(\lambda_0 - \varepsilon)+(\mu - \varepsilon)+(R-1)\theta(1 - H_2)]\ P_{1,R-1,0}+[(\lambda_0 - \varepsilon)]\ P_{0,R-1,0}$
$+[H_1(\lambda_0 - \varepsilon)]P_{1,R-2,0} = 0$ (10)

$-[(\lambda_1 - \varepsilon)+(r+1)\theta]P_{0,r+1,1}+(\mu - \varepsilon)P_{1,r+1,1}=0$ (11)

$-[(H_1(\lambda_1 - \varepsilon)+(\mu - \varepsilon)+(r+1)\theta(1 - H_2) + (r\alpha p)]\ P_{1,r+1,1} + [(\lambda_1 - \varepsilon)]\ P_{0,r+1,1} +$
$[H_1(\lambda_1 - \varepsilon)]P_{0,r+1,1}+(r+2)\theta\ P_{0,r+2,1} + [(r+2)\theta(1-H_2)+(r+1)\alpha p]\ P_{1,r+2,1} =0$ (12)

$-[(\lambda_1 - \varepsilon)+n\theta]P_{0,n,1}+(\mu - \varepsilon)P_{1,n,1}=0$ (13)

$-[H_1(\lambda_1 - \varepsilon)+(\mu - \varepsilon)+n\theta(1 - H_2) + (n-1)\alpha p]\ P_{1,n,1} + [(\lambda_1 - \varepsilon)]\ P_{0,n,1} +$
$[H_1(\lambda_1 - \varepsilon)]\ P_{1,n-1,1} + (n+1)\theta\ P_{0,n+1,1} + [(n+1)\theta(1-H_2)+n\alpha p]\ P_{1,n+1,1}=0,$
$n = r+2, r+3, \ldots\ldots.R-1$ (14)

$-[(\lambda_1 - \varepsilon)+R\theta]P_{0,R,1}+(\mu - \varepsilon)P_{1,R,1}=0$ (15)

$-[H_1(\lambda_1 - \varepsilon)+(\mu - \varepsilon)+R\theta(1 - H_2) + (R-1)\alpha p]\ P_{1,R,1} + [(\lambda_1 - \varepsilon)]\ P_{0,R,1} + [H_1(\lambda_1 - \varepsilon)]$
$P_{1,R-1,1} + [H_1(\lambda_0 - \varepsilon)]\ P_{1,R-1,0} + (R+1)\theta\ P_{0,R+1,1} + [(R+1)\theta(1-H_2)+R\alpha p]\ P_{1,R+1,1} =0$ (16)

$-[(\lambda_1 - \varepsilon)+n\theta]P_{0,n,1}+(\mu - \varepsilon)P_{1,n,1}=0$ (17)

$-[H_1(\lambda_1 - \varepsilon)+(\mu - \varepsilon)+n\theta(1 - H_2 + (n-1)\alpha p)]\ P_{1,n,1} + [(\lambda_1 - \varepsilon)]\ P_{0,n,1} +$
$[H_1(\lambda_1 - \varepsilon)]\ P_{1,n-1,1} + (n+1)\theta\ P_{0,n+1,1} + [(n+1)\theta(1-H_2)+n\alpha p]\ P_{1,n+1,1}=0,$
$n = R+1, R+2, \ldots\ldots\ldots K-1$ (18)

$-[(\lambda_1 - \varepsilon)+K\theta]P_{0,K,1}+(\mu - \varepsilon)P_{1,K,1}=0$ (19)

$-(\mu - \varepsilon)P_{1,K,1}+[H_1(\lambda_1 - \varepsilon)]\ P_{1,K-1,1} + [(\lambda_1 - \varepsilon)]\ P_{0,K,1} = 0$ (20)

Write $\gamma = [H_1(\lambda_0 - \varepsilon)]$ and $\delta = [H_1(\lambda_1 - \varepsilon)]$

From (1) to (4) we get,

$$P_{0,n,0}=\frac{\gamma^n \prod_{i=0}^{n-1}(\lambda_0-\varepsilon)+i\theta}{\prod_{i=1}^{n} i\theta(\mu-\varepsilon)+[i\theta(1-H_2)+(i-1)\alpha p][(\lambda_0-\varepsilon)+i\theta]} P_{0,0,0} \qquad 1\leq n\leq r \qquad (21)$$

$$P_{1,n,0}=\frac{\gamma^n}{\mu-\varepsilon}\prod_{i=1}^{n}\frac{[(\lambda_0-\varepsilon)+i\theta]}{i\theta(\mu-\varepsilon)+[i\theta(1-H_2)+(i-1)\alpha p][(\lambda_0-\varepsilon)+i\theta]}\quad P_{0,0,0} \tag{22}$$

From (5) to (8) we get,

$$P_{0,n,0}=\frac{\gamma^n\prod_{i=0}^{n-1}[(\lambda_0-\varepsilon)+i\theta]}{\prod_{i=1}^{n} i\theta(\mu-\varepsilon)+[i\theta(1-H_2)+(i-1)\alpha p][(\lambda_0-\varepsilon)+i\theta]}P_{0,0,0}-\left\{\frac{A_1}{A_2}\left(\left[\sum_{m-r}^{n-2}\gamma^{n-1-m}\quad\prod_{i=m+1}^{n}\frac{[(\lambda_0-\varepsilon)+i\theta]}{i\theta(\mu-\varepsilon)+[i\theta(1-H_2)+(i-1)\alpha p][(\lambda_0-\varepsilon)+i\theta]}\right]+1\right)\right\}\quad P_{0,r+1,1} \tag{23}$$

$$P_{1,n,0}=\frac{\gamma^n}{\mu-\varepsilon}\prod_{i=1}^{n}\frac{[(\lambda_0-\varepsilon)+i\theta]}{i\theta(\mu-\varepsilon)+[i\theta(1-H_2)+(i-1)\alpha p][(\lambda_0-\varepsilon)+i\theta]}\quad P_{0,0,0}-$$

$$\left\{\frac{A_1}{A_2(\mu-\varepsilon)}\left(\left[\sum_{m-r}^{n-1}\gamma^{n-1-m}\quad\prod_{i=m+1}^{n}\frac{[(\lambda_0-\varepsilon)+i\theta]}{i\theta(\mu-\varepsilon)+[i\theta(1-H_2)+(i-1)\alpha p][(\lambda_0-\varepsilon)+i\theta]}\right]\right)\right\}P_{0,r+1,1}$$

$$n = r+1, r+2, \ldots\ldots, R-1 \tag{24}$$

where

$A_1 = (r+1)\theta(\mu-\varepsilon)+[(r+1)\theta(1-H_2)+r\alpha p][(\lambda_1-\varepsilon)+(r+1)\theta]$

$A_2 = n\theta(\mu-\varepsilon)+[n\theta(1-H_2)+(n-1)\alpha p][(\lambda_0-\varepsilon)+n\theta]$

From (9) to (10) we get,

$P_{0,r+1,1}= \frac{A_3}{A_4}p_{0,0,0}$

Where

$$A_3=\frac{\gamma^R\prod_{i=0}^{R-1}[(\lambda_0-\varepsilon)+i\theta]}{\prod_{i=1}^{R-1} i\theta(\mu-\varepsilon)+[i\theta(1-H_2)+(i-1)\alpha p][(\lambda_0-\varepsilon)+i\theta]}P_{0,0,0}$$

$$A_4=-\left\{A_1\left(\left[\sum_{m-r}^{R-2}\gamma^{n-1-m}\quad\prod_{i=m+1}^{R-1}\frac{[(N-i)(\lambda_0-\varepsilon)+i\theta]}{i\theta(\mu-\varepsilon)+[i\theta(1-H_2)+(i-1)\alpha p][(N-i)(\lambda_0-\varepsilon)+i\theta]}\right]\right)\right\}P_{0,r+1,1} \tag{25}$$

From (11) to (14), we recursively derive,

$$P_{0,n,1} = \left\{\frac{A_1}{A_5}\left(\left[\sum_{m-r}^{n-2} \delta^{n-1-m} \prod_{i=m+1}^{n-1} \frac{[(\lambda_1-\varepsilon)+i\theta]}{i\theta(\mu-\varepsilon)+[i\theta(1-H_2)+(i-1)\alpha p][(\lambda_0-\varepsilon)+i\theta]}\right] + 1\right)\right\} P_{0,r+1,1} \quad (26)$$

$$P_{1,n,1} = \left\{\frac{A_1}{\mu-\varepsilon}\left(\left[\sum_{m-r}^{n-1} \delta^{n-1-m} \prod_{i=m+1}^{n} \frac{[(\lambda_1-\varepsilon)+i\theta]}{i\theta(\mu-\varepsilon)+[i\theta(1-H_2)+(i-1)\alpha p][(\lambda_1-\varepsilon)+i\theta]}\right]\right)\right\} P_{0,r+1,1}$$

n=r+1,r+2,……R-1,R (27)

where

$A_5 = n\theta(\mu-\varepsilon)+[n\theta(1-H_2)+(n-1)\alpha p][(\lambda_1-\varepsilon)+n\theta]$

A_1 is given by (23) and $P_{0,r+1,1}$ is given by (25)

From (15) to (20) we recursively derive,

$$P_{0,n,1} = \left\{\frac{A_1}{A_5}\left(\left[\sum_{m-r}^{R-2} \delta^{n-1-m} \prod_{i=m+1}^{n-1} \frac{[(\lambda_1-\varepsilon)+i\theta]}{i\theta(\mu-\varepsilon)+[i\theta(1-H_2)+(i-1)\alpha p][(\lambda_0-\varepsilon)+i\theta]}\right]\right)\right\} P_{0,r+1,1} \quad (28)$$

$$P_{1,n,1} = \left\{\frac{A_1}{\mu-\varepsilon}\left(\left[\sum_{m-r}^{R-1} \delta^{n-1-m} \prod_{i=m+1}^{n} \frac{[(\lambda_1-\varepsilon)+i\theta]}{i\theta(\mu-\varepsilon)+[i\theta(1-H_2)+(i-1)\alpha p][(\lambda_1-\varepsilon)+i\theta]}\right]\right)\right\} P_{0,r+1,1}$$

n=R+1, R+2,…K-1,K (29)

where A_1, A_5 and $P_{0,r+1,1}$ are given by (23), (25), (26).

Thus from (21) to (29), we find that all the steady state probabilities are expressed in terms of $P_{0,0,0}$.

2.2 Characteristics of The Model

The following system characteristics are considered and their analytical results are derived in this system.

- The probability P(0)that the system is in faster rate of primary arrivals with the server idle and busy.
- The probability P(1) that the system is in slower rate of primary arrivals with the server idle and busy.
- The probability $P_{0,0,0}$ that the system is empty.
- The expected number of customers in the system Ls_0, when the system is in faster rate of primary arrivals with the server idle and busy.
- The expected number of customers in the system Ls_1, when the system is in slower rate of primary arrivals with the server idle and busy.

The probability that the system is in faster rate of primary arrivals is

$$P(0) = \left[\sum_{n=0}^{r} P_{0,n,0} + \sum_{n=r+1}^{R-1} P_{0,n,0}\right] + \left[\sum_{n=0}^{r} P_{1,n,0} + \sum_{n=r+1}^{R-1} P_{1,n,0}\right] \quad (30)$$

The probability that the system is in slower rate of primary arrivals is,

$$P(1) = \left[\sum_{n=r+1}^{R} P_{0,n,1} + \sum_{n=R+1}^{K} P_{0,n,1}\right] + \left[\sum_{n=r+1}^{R} P_{1,n,1} + \sum_{n=R+1}^{K} P_{1,n,1}\right] \quad (31)$$

The probability $P_{0,0,0}$ that the system is empty can be calculated from the normalizing condition P(0) + P(1) =1.

$P_{0,0,0}$ is calculated from (30) and (31) .

Let L_s denote the average number of customers in the system, then we have

$$L_s = L_{s_0} + L_{s_1} \quad (32)$$

$$L_{s_0} = \left[\sum_{n=0}^{r} nP_{0,n,0} + \sum_{n=r+1}^{R-1} nP_{0,n,0}\right] + \left[\sum_{n=0}^{r}(n+1)P_{1,n,0} + \sum_{n=r+1}^{R-1}(n+1)P_{1,n,0}\right] \quad (33)$$

and

$$L_{s_1} = \left[\sum_{n=r+1}^{R} nP_{0,n,1} + \sum_{n=R+1}^{K} nP_{0,n,1}\right] + \left[\sum_{n=r+1}^{R}(n+1)P_{1,n,1} + \sum_{n=R+1}^{K}(n+1)P_{1,n,1}\right] \quad (34)$$

From (21) to (29) , (33) and (34) , we can calculate the value of L_s. The expected waiting time of the customers in the orbit is calculated as

$W_s = \frac{L_s}{\bar{\lambda}}$, Where $\bar{\lambda} = \lambda_0$ P(0) + λ_1P(1).

W_s is calculated from (30) to (32).

3.0 Numerical Illustrations

For various values λ_0, λ_1, μ, ϵ, θ, α , p, while r, R, K, H_1, H_2 are fixed values, computed and tabulated the values of $P_{0,0,0}$,P(0), P(1), L_s and W_s.

TABLE 1

λ_0	λ_1	μ	θ	ϵ	α	p=1-q	$P_{0,0,0}$
3	2	4	2	0.4	0.1	0.3	2.3560145×10^{-4}
3	2	4	2	1	0.1	0.3	1.4532178×10^{-4}
4	3	4	3	0.4	0.1	0.3	1.4602311×10^{-4}
4	3	4	2	0.4	0.1	0.3	4.0785335×10^{-5}
4	3	5	3	0.4	0.1	0.3	3.1167651×10^{-4}
4	3	4	3	0.4	0.1	0.3	8.3074121×10^{-5}
4	3	4	3	0.4	0.1	1	2.6349891×10^{-4}
3	2	4	2	0.4	0	0	2.5817178×10^{-4}
4	2	4	2	0.4	0.1	0.3	5.1127662×10^{-5}

TABLE 2

P(0)	P(1)	L_s	W_s
0.28701564	0.700852114	5.065371632	2.108041361
0.038153508	0.940624281	6.047660001	2.845447537
0.110616365	0.867161413	3.008270151	0.98503406
0.027676561	0.951101217	6.118758532	2.00616502
0.420180678	0.558607101	2.678737723	0.77304467
0.255356301	0.782421488	5.262622433	1.537306644
0.305041302	0.672736476	3.075472302	0.958800271
0.253661014	0.714126864	6.204065784	2.728770208
0.038732001	0.94004578	5.446472051	2.535753506

Conclusion

It is observed from the tables 1 and 2 that
when λ_0 increases keeping the other parameters fixed, $P_{0,0,0}$ and P(0) decrease but P(1), Ls and Ws increase. When λ_1 increases keeping the other parameters fixed, $P_{0,0,0}$ and P(0) decrease but P(1), Ls and Ws increase. When θ increases keeping the other parameters fixed, $P_{0,0,0}$ and P(0) increase but P(1), Ls andWs decrease. When θ increases keeping the other parameters fixed, $P_{0,0,0}$ and P(0) increase but P(1), Ls and Ws decrease. When μ increases keeping the other parameters fixed, $P_{0,0,0}$ and P(0) increase but P(1), Ls and Ws decrease. When p=1 keeping the other parameters fixed, $P_{0,0,0}$ and P(0) increase but P(1), Ls and Ws decrease. When $\alpha = 0$, $p = 0$ keeping the other parameters fixed, $P_{0,0,0}$ and P(0) decrease but P(1), Ls and Ws increase.

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